July 12, 1972

AD 745313

ARCTIC SITE SURVEY

Contract N00014-72-C-0335

Technical Report for Period April 1-June 30

Sponsored by Advanced Research Projects Agency
ARPA Order No. 1779/dtd. 11-16-71; Program Code No. 2N10

Scientific Officer

Director, Arctic Program Earth Sciences Division
Office of Naval Research
Department of the Navy
800 North Quincy Street
Arlington, VA 22217

Contract Effective - April 1, 1972   Contract Expiration - December 31, 1972

Contract Amount - $63638.00

Contractor:

Aerophysics Research Corporation
P. O. Box 187
Bellevue, Washington 98009
(206) 454-6927
Dr. T. S. Chow, Principal Investigator

7

1. SUMMARY

Part of Phase I of the work statement of Contract N00014-72-C-0335 has been completed. This includes the development of a ray tracing program for calculating the sound intensity from sources in the ocean with surface and bottom reflections. Salinity and temperature gradients have been taken into consideration to calculate the sound velocity and absorption loss. The sound intensity is calculated by energy considerations included in the ray tube of adjacent rays.

A formula has also been developed for calculating the intensity along a ray without considering the variation in the ray tube area. This involves the calculation of the radius of curvature of the wave front along the ray.

Future plans on ray tracing call for the development of techniques for calculating sound intensities in the caustic region and higher order corrections.

2. DISCUSSION

ENERGY ALONG A RAY PATH

Consider two-dimensional ray propagation in the (x,y) plane with sound velocity $c = c(x,y)$

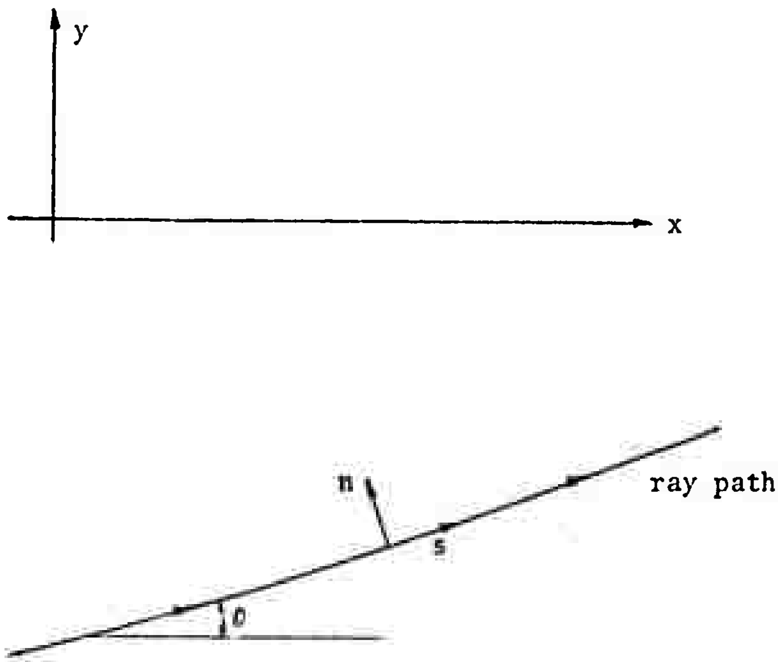

Fig. 1

Let s be the distance measured along a ray and let n denote the local orthogonal direction as shown. If θ is the slope angle of the ray, the $(\vec{s}, \vec{n})$ vectors are obtained by rotating the $(\vec{x}, \vec{y})$ vectors counterclockwise through θ.

The curvature $1/R$ is given by

$$\frac{1}{R} = \frac{d\theta}{ds} \tag{1}$$

Since the ray equations read

$$\begin{aligned} c\frac{d^2x}{ds^2} &= (c_x \frac{dx}{ds} + c_y \frac{dy}{ds})\frac{dx}{ds} - c_x \\ c\frac{d^2y}{ds^2} &= (c_x \frac{dx}{ds} + c_y \frac{dy}{ds})\frac{dy}{ds} - c_y \end{aligned} \tag{2}$$

It follows easily that

$$c\,\frac{d\theta}{ds} = c_x \sin\theta - c_y \cos\theta = -\frac{\partial c}{\partial n}$$

so that

$$\frac{1}{R} = \frac{\partial}{\partial n}\,(\ell n\ c) \tag{3}$$

The rays are orthogonal to surfaces of constant phase; for brevity, such surfaces will be termed *wave fronts*. Denote the local curvature of a wave front by $(1/R_w)$; taken as positive η the rays are diverging. The immediate purpose is to compute $d/ds(1/R_w)$.

For convenience, introduce an orthogonal curvilinear coordinate system with § constant on a ray and η constant on a wave front.

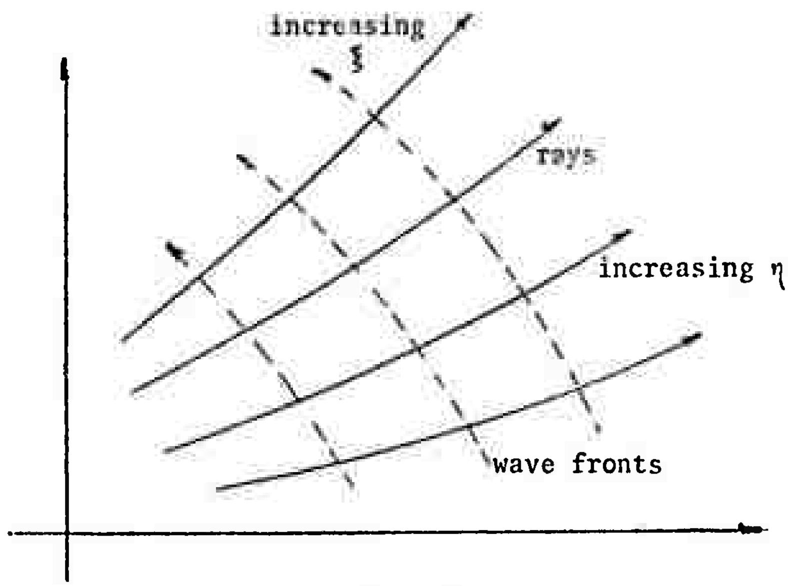

Fig. 2

Clearly,

$$\tan\theta = \frac{y_\eta}{x_\eta} = -\frac{x_§}{y_§} \tag{4}$$

and

$$\frac{1}{R} = \frac{\theta_\eta}{x_\eta}\cos\theta\ ,\ \frac{1}{R_w} = \frac{\theta_§}{y_§}\cos\theta \tag{5}$$

Thus,

$$\frac{d}{ds}\left(\frac{1}{R_w}\right) = \left(\frac{\theta_\xi}{y_\xi} \cos\theta\right)_\eta \frac{\cos\theta}{x_\eta}$$

$$= \frac{\cos\theta}{x_\eta y_\xi}\left[\theta_{\xi\eta}\cos\theta - \frac{\theta_\xi y_{\xi\eta}\cos\theta}{y_\xi} - \theta_\xi\theta_\eta\sin\theta\right] \tag{6}$$

Similarly,

$$\frac{d}{dn}\left(\frac{1}{R}\right) = \frac{\cos\theta}{x_\eta y_\xi}\left[\theta_{\xi\eta}\cos\theta - \frac{\theta_\eta x_{\xi\eta}\cos\theta}{x_\eta} - \theta_\xi\theta_\eta\sin\theta\right] \tag{7}$$

combining, we obtain

$$\frac{d}{ds}\left(\frac{1}{R_w}\right) = \frac{d}{dn}\left(\frac{1}{R}\right) + \frac{\cos\theta}{x_\eta y_\xi}\left(\frac{x_{\xi\eta}}{R} - \frac{y_{\xi\eta}}{R_w}\right) \tag{8}$$

Next, differentiate the first of Equations (4) with respect to ξ and the second with respect to η ; then solve the resulting equations for $y_{\xi\eta}$ and $x_{\xi\eta}$ to obtain (with the help of Equations (5))

$$\begin{aligned} y_{\xi\eta} &= \frac{x_\eta y_\xi}{\cos\theta}\left(\frac{1}{R_w} - \frac{\tan\theta}{R}\right) \\ x_{\xi\eta} &= \frac{x_\eta y_\xi}{\cos\theta}\left(-\frac{1}{R} - \frac{\tan\theta}{R_w}\right) \end{aligned} \tag{9}$$

Substitution into Equation (8) now gives the desired result:

$$\frac{d}{ds}\left(\frac{1}{R_w}\right) = \frac{d}{dn}\left(\frac{1}{R}\right) - \frac{1}{R^2} - \frac{1}{R_w^2} \tag{10}$$

To obtain a convenient formula for d/dn(1/R), we observe that Equation (3) yields the following.

$$\frac{d}{dn}(\frac{1}{R}) = \frac{d}{dn}[(\ell n\ c)_x \sin\theta - (\ell n\ c)_y \cos\theta]$$

$$= [(\ell n\ c)_x \cos\theta + (\ell n\ c)_y \sin\theta] \frac{d\theta}{dn}$$

$$- [(\ell n\ c)_{xx} \sin^2\theta - 2(\ell n\ c)_{xy} \sin\theta \cos\theta + (\ell n\ c)_{yy} \cos^2\theta]$$

$$= \frac{1}{R_w} [\frac{d}{ds}(\ell n\ c)] - \frac{d^2}{dn^2}[\ell n\ c] \qquad (11)$$

where d^2/dn^2 is defined in the "straight line" sense via the progression between the last two lines of Equation 11. Carrying out the differentiation of (ℓn c) gives the alternative form

$$\frac{d}{ds}(\frac{1}{R_w}) = \frac{1}{cR_w}\frac{dc}{ds} - \frac{1}{c}\frac{d^2c}{dn^2} - \frac{1}{R_w{}^2} \qquad (12)$$

where

$$\frac{d^2c}{dn^2} = c_{xx} \sin^2\theta - 2c_{xy} \sin\theta \cos\theta + c_{yy} \cos^2\theta \qquad (13)$$

Suppose now that the source is located at some point on the (negative) y axis. At each point along a ray we know the range x, the slope angle θ, and the wave front curvature $1/R_W$ (as a result of Equation (12)). Let F denote the intensity (energy rate per unit area) along a ray. It now follows from simple geometry that

$$-\frac{1}{F}\frac{dF}{ds} = \frac{\cos\theta}{x} + \frac{1}{R_W} \qquad (14)$$

If A(x, y) denotes the acoustic attenuation in the water, then the final equation becomes

$$-\frac{1}{F}\frac{dF}{ds} = \frac{\cos\theta}{x} + \frac{1}{R_W} + A(x,y) \qquad (15)$$

In a program, it is worthwhile to record each term in Equation (15) separately; thus, F is divided into the three terms F_R, F_W, and F_A .

via

$$F = F_R \cdot F_W \cdot F_A$$

so that

$$\frac{1}{F_R}\frac{dF_R}{ds} = \frac{\cos\theta}{x} = \frac{1}{x}\frac{dx}{ds}$$

$$\frac{1}{F_W}\frac{dF_W}{ds} = \frac{1}{R_W} \qquad (16)$$

$$\frac{1}{F_A}\frac{dF_A}{ds} = A(x, y)$$

The first of these equations yields

$$F_R = \frac{\text{const.}}{x}$$

when the constant is chosen to be the intensity of the source at unit distance (i.e., $s = 1$), along the chosen ray; thus, F_W and F_A are each unity at that point.

A knowledge of F_W has an interesting physical interpretation. Consider two rays emanating from the source at angle θ_0, at an incremental angle $d\theta_0$ apart. At unit distance from the source, $F_W = 1$. At a terminal point x_t, where the slope angle is θ_t and the value of F_W is F_{Wt}, we can use the fact that F_W is inversely proportional to the normal spacing dn between adjacent rays to write

$$(1)(1)d\theta_0 = F_{Wt}|dn| \qquad (17)$$

so that $|dn/d\theta_0| = 1/F_{Wt}$